G. Spontaneous Emission

$$A_{21} = \frac{\hbar\omega^3}{\pi^2 c^3} B_{(21)\text{ or }(12)} \text{ (Einstein)}; \quad \lambda_{1\to 2} = \overbrace{\frac{\pi e^2}{3\varepsilon_0 \hbar^2} |r_{21}|^2}^{B_{12}\,(=B_{21})} \cdot U(\omega_{21}) \text{ (QM)}$$

- "Borrowing" Einstein's result

$$\boxed{A_{21} = \frac{\omega^3}{3\pi\varepsilon_0 c^3 \hbar} e^2 |r_{21}|^2} \quad (40)$$

Obtained something from nothing!

(Einstein came to help!)

- Same selection rules ($\because \sim |r_{21}|^2$)
- Eq. (40) is a QM result
  - Einstein didn't know
  - Schrödinger QM can't treat "vacuum"

[But when they "meet", Eq. (40) provides a way to get A and $1/A$]

- $A \sim \frac{1}{\text{time}}$

Prob. per atom per time

Lifetime of Excited States

- Excited state could have short lifetime τ because
  - there is (are) lower state(s) with allowed transitions ($\because |r_{21}|^2$)
  - excited state is high in energy ($\because \omega^3$)
  - many lower states to go to

Short lifetime

$\tau \sim 10^{-9}\,s - 10^{-8}\,s$

H-atom: $2p \rightarrow 1s$ ($\sim 10^{-9}\,s$)

Long Life time

$\tau \sim 10^{-3}\,s$

[Metastable state] (important for operation of LASER)

Life time is related to spontaneous emission

- Let $N_2(t)$ be number of excited atoms (at state "2") at time t

  $N_2(t)$ drops due to spontaneous emission (No Stimulation)

  Recall: $A_{(21)}$ or A is prob. per atom per unit time to decay to a lower state "1"

  $$\therefore \frac{dN_2(t)}{dt} = -A\, N_2(t)$$

  Einstein's A coefficient

  $$\Rightarrow N_2(t) = N_2(0)\, e^{-At} \equiv N_2(0)\, e^{-t/\tau} \quad (\tau = \text{life time})$$

  $$\therefore \boxed{\tau = \frac{1}{A}} \quad (41)$$

  Einstein's A-coefficient inversed is excited state's lifetime

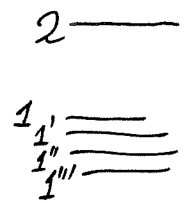
  (really getting many things from nothing!)

For state "2" having only state "1" to decay to, A_{21} is given by (40).

Thus
$$\boxed{\tau = \frac{3\pi\varepsilon_0 c^3 \hbar}{\omega^3} \frac{1}{e^2 |r_{21}|^2}} \quad (42) \qquad \text{(Magical!)}$$

- Appreciation: QM provides a formula to calculate τ through evaluating an integral r_{21} (recall $\omega = \frac{E_2 - E_1}{\hbar}$)
- $\sim \frac{1}{\omega^3} \Rightarrow \underbrace{\text{highly excited states}}_{\text{Very hard to study! (A hot research topic, see "Rydberg Atoms")}}$ have short τ
- state with lower state (allowed) to go $\Rightarrow |r_{21}|^2$ not small $\Rightarrow$ short τ
- state with no lower state to go $\Rightarrow$ long τ (meta stable state)

What if State "2" has many lower states to go?



$$\frac{dN_2(t)}{dt} = -A_{21}N_2(t) - A_{21'}N_2(t) - A_{21''}N_2(t) \cdots$$

$$\therefore \quad \tau_2 = \frac{1}{A_{21} + A_{21'} + A_{21''} + A_{21'''} + \cdots}$$

this is τ of state 2

- More states to go $\Rightarrow$ shorter τ
- Biggest A dominates

OR

$$\frac{1}{\tau_2} = A_{21} + A_{21'} + A_{21''} + A_{21'''} + \cdots$$

$$= \frac{1}{\tau_{21}} + \frac{1}{\tau_{21'}} + \frac{1}{\tau_{21''}} + \frac{1}{\tau_{21'''}} + \cdots$$

due to $2 \rightarrow 1$ due to $2 \rightarrow 1'$ due to $2 \rightarrow 1'''$ (each can be handled by QM)

Key Point: life time is related to spontaneous emission through the A-coefficient

Final Remark

- Method and Results in "LMI" module are applicable to light (absorption, emission) interacting with matter
    - Atoms (transitions between atomic states)
    - Molecules (transitions between $\underbrace{\text{molecular states}}_{\text{[electronic, vibrational, rotational]}}$)
    - Solids (transitions from valence band to conduction band in semiconductors)

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